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LETTER TO THE EDITOR

Self-amplification of turbulent 3D vorticity field and 2D vorticity gradient

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Abstract. The effects of self-amplification of vorticity field in 3D turbulence and vorticity gradient in 2D turbulence are considered. The conditionally averaged tensor of strain rates (with fixed vorticity) is obtained for 3D turbulence. The corresponding tensor (with fixed vorticity gradient) is obtained for 2D turbulence. These results are discussed in the context of observations and direct numerical simulations of turbulent flows. In particular, the appearance of 'vortex strings' in 3D turbulent flows is in accord with the obtained formula (8). The presented method is quite general and can be applied to a variety of physical systems with strong interaction.

The internal dynamics of strong interactions in turbulent flows is better described in terms of local characteristics of motion [1]. Local characteristics must have a mechanism of self-amplification and should not depend directly on the sources of energy. For 3D turbulence the local characteristic of motion is the vorticity field [2–4] and self-amplification is the well known effect of stretching of vortex filaments. For 2D turbulence the local characteristic of motion is the vorticity gradient (VG) [4, 5] and self-amplification is the compression of fluid element in the direction of VG. We use the concept of self-amplification because in both cases the tensor of strain rates is expressed in terms of local characteristics.

The hierarchy of 'kinetic' equations for a 3D vorticity field was obtained in [3] from the Navier–Stokes equations. These are the equations for n-particle (Lagrangian) and n-point (Eulerian) probability distributions of vorticity field. It turns out to be very useful to express the first equation of this hierarchy in terms of a conditionally averaged vorticity field $\bar{\Omega}_i$ on a distance $\boldsymbol{r}$ from the point $\boldsymbol{x}$ with fixed vorticity $\boldsymbol{\omega}$ [4]

$$\bar{\Omega}_i(\boldsymbol{r},\boldsymbol{\omega}) = p_1^{-1}(\boldsymbol{\omega}) \int \omega_i' p_2(\boldsymbol{\omega}',\boldsymbol{\omega},\boldsymbol{r})\,\mathrm{d}\boldsymbol{\omega}' \qquad \boldsymbol{r} = \boldsymbol{x}' - \boldsymbol{x}. \tag{1}$$

Here p_1 and p_2 are one-point and two-point probability distributions of vorticity. For statistically non-homogeneous flow all characteristics can depend slowly on position $\boldsymbol{x}$. We will call the Fourier transformation of (1) the conditional spectrum of vorticity

$$\tilde{\Omega}_i(\boldsymbol{k},\boldsymbol{\omega}) \doteq \frac{1}{(2\pi)^3} \int \exp\{-\mathrm{i}\boldsymbol{k}\boldsymbol{r}\}\bar{\Omega}_i(\boldsymbol{r},\boldsymbol{\omega})\,\mathrm{d}\boldsymbol{r} \tag{2}$$

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where $\boldsymbol{k}$ is the wavenumber vector.

We express the space derivative of velocity in terms of vorticity for incompressible flow

$$\frac{\partial v_i(\boldsymbol{x})}{\partial x_k} = -\frac{1}{(2\pi)^3}\epsilon_{ijm}\int n_k n_j \exp\{-\mathrm{i}\boldsymbol{k}(\boldsymbol{x}'-\boldsymbol{x})\}\omega_m(\boldsymbol{x}')\,\mathrm{d}\boldsymbol{k}\,\mathrm{d}\boldsymbol{x}'. \tag{3}$$

Here ϵ_{ijm} is the unit axisymmetric tensor and $n_i = k_i k^{-1}$ is the unit wavenumber vector. The conditional averaging of (4) with fixed ω at point $\boldsymbol{x}$ gives

$$\overline{\frac{\partial v_i}{\partial x_k}} = -\epsilon_{ijm}\int n_k n_j \tilde{\Omega}_m(\boldsymbol{k},\omega)\,\mathrm{d}\boldsymbol{k} \tag{4}$$

where bar means conditional averaging. The same procedure can be applied directly to the Navier–Stokes equations (written in terms of vorticity field). For locally isotropic turbulence the result is [4]

$$\omega_k\epsilon_{ijm}\int n_m n_k \tilde{\Omega}_j(\boldsymbol{k},\omega)\,\mathrm{d}\boldsymbol{k} = \nu\int k^2\tilde{\Omega}_i(\boldsymbol{k},\omega)\,\mathrm{d}\boldsymbol{k} \tag{5}$$

ν being molecular viscosity. For non-homogeneous and decaying turbulence additional terms are of order of $R^{-1/2}$ (R being Reynolds number) [4]. Equation (5), which is equivalent to the first equation of the above mentioned hierarchy [3] represents the statistical balance between vortex stretching and viscous smoothing. By multiplying (5) on ω_i and averaging over ω, we get the enstrophy balance

$$\left\langle\frac{\partial v_i}{\partial x_k}\,\omega_i\,\omega_k\right\rangle = \nu\left\langle\left(\frac{\partial\omega_i}{\partial x_k}\right)^2\right\rangle \tag{6}$$

where $\langle\ \rangle$ means unconditional averaging. The analysis of equation (5) and some solutions are presented in [4].

By symmetrization of (4), we get the conditionally averaged deformation tensor

$$\overline{D_{ik}} = \frac{1}{2}\left(\overline{\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}}\right) \qquad \overline{D_{ii}} = 0. \tag{7}$$

This symmetric tensor depends only on the vector ω_i and we have only two symmetric tensors $\omega_i\omega_k$ and δ_{ik}. Taking into account incompressibility and (4), we have

$$\overline{D_{ik}} = \tfrac{1}{2}\,\alpha(\omega)\,(3\sigma_i\,\sigma_k - \delta_{ik}) \qquad \sigma_i = \omega_i\omega^{-1} \quad \omega = |\omega| \tag{8}$$

$$\alpha(\omega) = -\epsilon_{ijm}\,\sigma_i\int n_j\,\tilde{\Omega}_m\,\mu\,\mathrm{d}\boldsymbol{k} \qquad \mu = \sigma_i\,n_i\,. \tag{9}$$

Function $\alpha(\omega)$, principally, can be negative for some ω, but with condition (6)

$$\langle\alpha(\omega)\,\omega^2\rangle = \nu\left\langle\left(\frac{\partial\omega_i}{\partial x_k}\right)^2\right\rangle > 0\,. \tag{10}$$

For some solutions [4] of equation (5), expression (9) does not depend on ω. *In this* case, from (10) we get

$$\alpha = \frac{\nu^2}{\epsilon}\left\langle \left(\frac{\partial \omega_i}{\partial x_k}\right)^2 \right\rangle \qquad \nu\langle\omega^2\rangle = \epsilon \tag{11}$$

ϵ being the mean rate of energy dissipation.

Formula (8) tells us that the maximum absolute rate of strain is along the direction of $\boldsymbol{\omega}$. Two other eigenvalues are equal. This exact result for locally isotropic turbulence, generally speaking, does not undermine the findings from numerical experiment [6] about the most probable orientation of vorticity along the axis with intermediate strain rate. These are two different sets of statistical characteristics. In the future, it will be interesting to calculate from direct numerical simulations the conditionally averaged deformation tensor with fixed vorticity. The conditionally averaged vorticity field (1) can also be obtained from numerical experiments and compared with (8), (9) and with solutions, presented in [4]. Formula (8) (with typical $\alpha(\omega) > 0$, see (10)) is in accord with appearance of 'vortex *strings' in* 3D *turbulent flows (see observations* in [7] and references therein for numerical experiments).

For 2D turbulent flow the local characteristic is the vorticity gradient (VG)

$$s_i = \frac{\partial \omega}{\partial x_i} \qquad \omega = \epsilon_{ij}\,\frac{\partial v_j}{\partial x_i}. \tag{12}$$

Here ϵ_{ij} is unit axisymmetrix 2D tensor, ω is now the only one component of vorticity, perpendicular to the plane of motion. Formula (8) in the 2D case takes the form

$$\overline{D_{ik}} = \beta(s)\,(2\gamma_i\,\gamma_k - \delta_{ik}) \qquad \gamma_i = s_i\,s^{-1} \qquad s = |\boldsymbol{s}|\,. \tag{13}$$

Here bar now means conditional averaging with fixed $\boldsymbol{s}$. For the function $\beta(s)$ we get an expression, analogous to (9)

$$\beta(s) = \mathrm{i}\epsilon_{ij}\gamma_i \int k^{-1}\theta\tilde{S}_j(\boldsymbol{k},\boldsymbol{s})\,\mathrm{d}\boldsymbol{k} \qquad \theta = \gamma_i n_i \tag{14}$$

$$\tilde{S}_i(\boldsymbol{k},\boldsymbol{s}) = \frac{1}{(2\pi)^2}\int \exp\{-\mathrm{i}\boldsymbol{k}\boldsymbol{r}\}\bar{S}(\boldsymbol{r},\boldsymbol{s})\,\mathrm{d}\boldsymbol{r}\,. \tag{15}$$

Here $\bar{S}$ is the conditionally averaged VG *on a distance $\boldsymbol{r}$ from the point with fixed $\boldsymbol{s}$,* $\tilde{S}$ is the conditional spectrum of VG. The equation for $\tilde{S}$ has the form [4, 5]

$$\mathrm{i}\epsilon_{jm}s_m \int k^{-1}n_i\tilde{S}_j(\boldsymbol{k},\boldsymbol{s})\,\mathrm{d}\boldsymbol{k} = \nu\int k^2\tilde{S}_i(\boldsymbol{k},\boldsymbol{s})\,\mathrm{d}\boldsymbol{k}. \tag{16}$$

Numerical simulations of 2D turbulence can be compared with (13), (14), as well as with solutions of equation (16), presented in [4, 5]. For 2D turbulence numerical simulation can be done with higher Reynolds number than for 3D turbulence and comparison with the results presented above is easier. The method presented above is quite general and can be applied to a variety of physical systems with strong interaction, including *strongly turbulent plasma.*

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